MCA Study Skills
Statistics & Probability Unit

Minnesota Standards: Probability (Sub-Strand IV.B)

Lesson 4: Counting Techniques
~Factorial
~Permutations
~Combinations
~Word Permutations

Minnesota Standards: Probability (Sub-Strand IV.B)

Factorial, denoted "!" is the product of every number including the number itself on down to 1.

6! = 6 x 5 x 4 x 3 x 2 x 1 = 720

3! = 3 x 2 x 1 = 6

0! = 1

Example: Find the following

(A) $\frac{8!}{5!}$

Solution

$= \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!}}$

= 8 x 7 x 6

= 336

(B) $\frac{10!}{7!}$

Solution

$\frac{10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}}$

= 10 x 9 x 8

= 720

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Permutations and Combinations are used when counting the number of outcomes in a problem.

Permutations are used when the specific order in which the problems are counted is important or matters.

Combinations are used when the specific order in which the problems does not matter.

Permutations
(Order is important)

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

n= total objects
r = objects desired

Combinations
(Order not important)

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

n= total objects
r = objects desired

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Permutations & Combinations

The Multiplication Rule For Counting

When a problem is based around the order being important (permutation), the counting principle can also be used in this case. The multiplication principle says to multiply each individual outcome together in order to determine the total possible combinations for a desired scenario.

Example 1

How many possible ways can 6 people be seated in 6 chairs?

Solution: 6 x 5 x 4 x 3 x 2 x 1 = 720 different ways

Example 2:

How many different license plates are possible if the first 3 spots are letters and the last 3 are numbers?

Solution: 26 x 26 x 26 x 10 x 10 x 10 = 17576000

How many different license plates are there in the problem above if only vowels can be used and only even numbers can be used?

Solution: 5 x 5 x 5 x 5 x 5 x 5 = 15625

Example 3:

Radio stations go by a 4-letter call sign (KCLD, KASM). How many different 4-letter call signs are possible if the first letter must be a K or L and the last letter must not be a X,Y, or Z. Repeating is allowed.

Solution: 2 x 26 x 26 x 23 = 31096

How many 4-letter call signs are possible in the problem above if letters cannot be repeated?

Solution 2 x 24 x 23 x 22 = 24288

Example 4:

A basketball coach starts a traditional line-up of 2 Guards, 2 Forwards, and 1 center. How many line-ups are possible if there are 5 Guards, 4 forwards, and 2 centers on the team?

Solution: 5 x 4 x 4 x 3 x 2 = 480

Example 5:

In 5 card stud Poker, how many different hands are possible if you are looking to get a flush (5 of the same suit)?

Solution: 52 x 12 x 11 x 10 x 9 = 617760

How many different ways can you get a full house?

Solution: 52 x 3 x 2 x 48 x 3=44928

How many different ways can you get 3 Jacks?

Solution: 4 x 3 x 2 x 48 x 44 = 50688

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Permutations & Combinations

Find the number of permutations or combinations for each problem below:

1.) Erik has 11 shirts to wear to school. How many ways can he choose a different shirt to wear on Monday, Tuesday, Wednesday, Thursday, and Friday?

Solution: This problem is a permutation: $_{11}P_5 = \frac{11!}{(11-5)!} = \frac{11!}{6!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6!} = 55440$

2.) Angelo's Pizza offers 10 different pizza toppings. How many different combinations can be made of pizzas with four toppings?

Solution: this problem is a combination: $_{10}C_4 = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6! \times 4!} = 210$

3.) At a restaurant, there are 8 tables open up at the same time. How many different ways can the hostess seat the next four families to be seated?

Solution: This is a permutation: You could also use the counting principle here: 8 x 7 x 6 x 5 = 1680

4.) A fast food restaurant has 8 kinds of sandwiches, 3 kinds of French fries, and 5 kinds of soft drinks. how many combinations of meals could you order if you ordered a sandwich, fries, and a drink?

Solution: This is a combination: Use the counting principle: 8 x 3 x 5 = 120

5.) Sandi's couch can hold 3 people. How many different ways can she and 2 other friends sit on the couch?

Solution: this is a permutation: Use the counting principle: 3 x 2 x 1 = 6

6.) Raymond has 7 baseball caps, 2 jackets, 10 pairs of jeans, and 2 pairs of sneakers. How many combinations of the 4 items can he make?

Solution: This is a combination: Use the counting principle: 7 x 2 x 10 x 2 = 280

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Word Permutations

The total number of ways that a letter arrangement can be rearranged can be found by using the following formula:

$$\frac{n!}{(\#\text{ of times a letter occurs more than once})!(\#\text{ of times a letter occurs more than once})!\ldots}$$

Where n is the number of letters in the word

Example 1:

How many different letter arrangements can be made from the following words:

a.) Algebra **b.) Minnesota**

Solution: $\frac{7!}{2!} = 2520$ **Solution:** $\frac{9!}{2!} = 181440$

c.) Mississippi **d.) Illies**

Solution: $\frac{11!}{4!4!2!} = 34650$ **Solution:** $\frac{6!}{2!2!} = 180$